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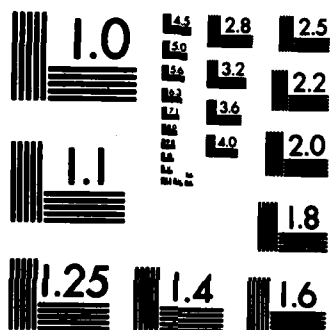
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"PAIRWISE BALANCED" LATIN SQUARES SHOULD ALWAYS BE  
USED FOR WITHIN-SUBJECTS DESIGNS

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Block 20 (Abstract) - Continued

↙ administered in each serial position of the treatment sequence. The present paper presents a discussion of a novel technique for the generation of a subset of Latin Squares that control for two additional features that are seen to be important in many research situations, i.e., pairwise priority and distance. Such Latin Squares are referred to as 'pairwise balanced' Latin Squares. The relative advantages of using such Latin Squares in repeated measures designs are discussed. ↗

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## Abstract

The use of repeated measures designs in many areas of psychological research has prompted concern for the potential confounds inherent in the interpretation of within-subjects effects. Of the solutions proposed for this problem, the most commonly adopted strategy is the use of Latin Square counterbalancing orders for treatment presentation. Traditional Latin Square designs ensure that each of the experimental treatments appear equally often in each serial position of the treatment sequence. The present paper presents a technique for generating a subset of Latin Squares that control for two additional characteristics of treatment sequence. Pairwise priority refers to the proportion of times that for any given treatment pair,  $x$  and  $y$ , Treatment  $x$  precedes Treatment  $y$ . A subset of Latin Squares exists for which this proportion is .5 for all treatment pairs. Pairwise distance refers to the number of other treatments that come between treatment pair  $x$  and  $y$  in the treatment sequence. A subset of Latin Squares exists that partially controls for the distribution of distances across all treatment pairs. The subset of Latin Squares that controls for both pairwise priority and pairwise distance are referred to as 'pairwise balanced' Latin Squares.

Within-subject designs are being used with increasing frequency in psychological research. For example, Poulton (1982) compared the types of experimental designs employed in research reported in the Journal of Experimental Psychology in September 1972 and 1979 and found that the ratio of within-subject to between-subjects designs had increased from 1.7: 1 to 7.3: 1. This increase has been accompanied by commentary and analysis concerning the adequacy of such within-subject designs to provide unambiguous tests of experimental hypotheses (e.g., Greenwald, 1976; Poulton, 1973, 1974, 1982; Rothstein, 1974). The central concerns embodied in these commentaries relate both to matters of experimental procedure and the proper interpretation of experimental results. Of course, the two are interrelated in that improvements in procedure often serve to lessen interpretive cautions.

The most efficient procedure for dealing with the interpretive problems of within-subject designs involves the use of Latin Square counter-balanced orders for treatment presentation (e.g., Lindman, 1974; Myers, 1979; Winer, 1972). The purpose of the present paper is to address the adequacy of traditional Latin Square selection criteria. The traditional criteria focus exclusively on guaranteeing that all treatments appear equally often in all serial order positions of the treatment presentation sequence. In this paper, we argue that two additional criteria should always be invoked when selecting a Latin Square, namely the criteria of "pairwise priority" and "pairwise distance".

Within-subject Treatments and Latin Square Designs

In within-subject designs, each subject is exposed to all of the experimental treatments. This type of design is often preferred because: a) it allows for greater economy in subject utilization, b) it often serves to increase the statistical power of hypothesis tests, and c) it is often a more ecologically valid way of examining specific research hypotheses (e.g., Greenwald, 1976, but see Poulton, 1982).

Although preferred for these reasons, within-subject designs are also encumbered by procedural weaknesses that often leave the research open to plausible alternative explanations for the obtained experimental results. Chief among these potential confounds are those associated with order or sequence effects, practice and/or fatigue effects, and the residual effects (also referred to as transfer, carry-over or range effects) of treatments (e.g., Campbell & Stanley, 1966; Carlsmith, Ellsworth & Aronson, 1976; Christensen, 1980; Crano & Brewer, 1973; D'Amato, 1970; Greenwald, 1976; Poulton, 1973, 1974, 1982). Several general types of solutions have been suggested in attempts to take such potential sequence effects into account. The most commonly adopted procedure involves the use of Latin Square counterbalancing of treatment orders.

Traditional criteria for Latin Square selection.

Latin Squares control serial position effects by ensuring that each treatment appears equally often in each order position.

Traditional selection criteria focus on random selection from the population of all possible squares. Consider the guidelines outlined by Winer (1972). He suggests that one first randomly select a standard square from such sources as Fisher & Yates (1953) or Cochran & Cox (1957). Next, the columns and rows are randomly reordered. Winer provides an example for the 4X4 Latin Square case. Starting with the square on the left of Table 1, Winer reordered the columns and rows according to the random number sequences 2, 4, 1, 3 and 3, 4, 1, 2 producing the square on the

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Insert Table 1 about here

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right of Table 1. In Latin Square designs such as this, the columns refer to the serial order of treatments (a within-subjects factor) and the rows refer to subject types (a between-subjects factor).

In practice, many investigators bypass the recommended procedure and generate their own square in the simplest manner possible. This can be done by randomly assigning treatments to positions in the first row of a square and then cyclically permuting each subsequent row. To do this, one simply takes the last condition of the first row and puts it in the first position of the second row. All other treatments are then shifted accordingly one position to the right. By coincidence, the recommended square produced by Winer's randomization procedure (see Table 1) yielded such a cyclical square. This can be seen most easily by transposing rows

two and three in Winer's recommended square. Even Cochran and Cox (1957, p. 145-146) revert to the use of cyclical squares when  $n$  is greater than 6. Such cyclical squares, unfortunately, always introduce pairwise biases, and therefore should always be avoided for counterbalancing in repeated measures designs.

#### New Criteria for Latin Square Selection

It is clear that the cyclically generated squares, as well as those generated in the traditional manner, all satisfy the selection criterion of ensuring that each treatment appears in each of the four treatment serial positions. These commonly used squares, however, fail to explicitly control for two other features of treatment sequence that can affect the interpretation of within-subject treatment differences, i.e., pairwise priority and pairwise distance.

Pairwise priority refers to the proportion of times (across all subject types) that "treatment  $x$ " precedes "treatment  $y$ ". When that proportion is exactly .5, this means that  $x$  precedes  $y$  as often as  $y$  precedes  $x$ . For example, note that in the condition pair of 0, 1 in the recommended square of Table 1 the proportion is exactly .5 (or  $2/4$ ), whereas for the pair 0, 2 the proportion is .75 (or  $3/4$ ). A subset of squares exists in which all pairs have exactly a .5 probability. Such squares are considered to be balanced for pairwise priority.

Pairwise distance refers to the number of other treatments (counting forward or backward from the numerically smaller member

of the treatment pair to the larger) occurring between a particular pair of treatments,  $x$  and  $y$ . For example, in the first line of the recommended square of Table 1, there is a distance of two units between the 1, 3 condition pair. Ideally, one would want to exactly control the distribution of distances over the entire design for all condition pairs. Unfortunately no Latin Squares exist that provide such a control.

There are two features of the distributions of pairwise distances that can be controlled within a single square. The first is the proportion of pairs that are contiguous (i.e., the proportion of times, over all subject types, that a particular pair has a distance of zero). Note in the recommended square of Table 1 the pair 0, 2 are contiguous three of four times, whereas the pair 0, 1 are never contiguous. Of the six pairs in this square, four have at least one contiguous occurrence and two (0, 1 and 2, 3) have no contiguous subject types. A subset of Latin Squares exists in which all condition pairs have exactly two subject types with zero distance. In such squares, the proportion of contiguous pairs is constant for all possible pairs.

There is a second feature of the distribution of distances between pairs that can be controlled. In squares balanced for pairwise priority, it is possible to obtain directional symmetry. One can exactly match the distribution of distances for subject types in which Condition  $x$  precedes Condition  $y$  with the distribution obtained when Condition  $x$  follows Condition  $y$ . Thus directional

differences in priority will not be confounded with distance. Latin Squares that control for these two features of distance (directional symmetry and proportion of contiguous pairs) are considered balanced for distance.

#### Construction of Pairwise Balanced Latin Squares

The existence of subsets of Latin Squares incorporating features related to the present concerns has been acknowledged in the past (e.g., "diagram" balanced designs of Wagenaar, 1969, and designs "balanced for the estimation of residual effects", as discussed by Alimena, 1962; Cochran & Cox, 1959; and Williams, 1949). However, these earlier authors have not addressed the special implications of these squares for counterbalancing in psychological research. We have also been able to improve on the procedures presented in this earlier work for identifying acceptable squares (Isaac, McCann & Ostrom, 1983). For example, our procedure for even number designs generates more squares than does Williams' (1949) procedure, and includes squares equivalent to those generated by Alimena (1962) and Wagenaar (1969). Reports of these earlier procedures are absent from many books on statistics (e.g., Winer, 1972; Meyers, 1979) and research design (e.g., Crano & Brewer, 1973; Murphy & Puff, 1982) that appear in the psychological literature.

Since procedures for generating pairwise balanced pairs are available elsewhere, we will not repeat them here. Instead we have prepared tables that summarize squares ranging in size from three to sixteen. Most repeated measures research in psychology

involves designs in that range. We should also note at this point that no single generation procedure exhaustively represents the entire population of pairwise balanced squares for any given  $n$  (see Isaac, McCann, & Ostrom, 1983).

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Insert Tables 2 and 3 about here

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The entries in Tables 2 resulted from applying procedures referred to in Isaac et al. (1983). Those in Table 3 were initially developed by Williams (1949). We suspect that even more could be produced through trial and error (see Wagenaar, 1969, and Denes & Keedwell, 1974).

#### How to use Tables 2 and 3

Tables 2 and 3 do not contain the full Latin Squares; rather, they provide only the first line of the one or more pairwise balanced squares given for each size  $n$ . This first line corresponds to "Subject Type I" as described in Table 1. The lines corresponding to the remaining Subject Types are easily produced in the manner described below.

1. Determine the size of Latin Square needed for the research design. The size ( $n$ ) corresponds to the number of treatments in the repeated measures experiment.
2. Select a first line from Table 2 or 3 that corresponds to  $n$ . If more than one is listed in the table, select one randomly.
3. Generate the remaining rows (or Subject Types) of the square. Successive rows are produced by adding one (in modular

arithmetic) to each entry in the previous row. As an illustration, consider the square of  $n = 4$  in Table 2. The second, third, and fourth rows are 1, 2, 0, 3; 2, 3, 1, 0; and 3, 0, 2, 1, respectively.

4. Randomly assign experimental treatments to the  $n$  numbers in the resulting square. Note that this means when an  $n$  by  $n$  square is generated, it forms the basis of  $n!$  squares of experimental treatments.

5. Randomly assign subjects to rows of the square, insuring that an equal number of subjects are assigned to each.

#### Odd size squares

Complications arise when an experiment involves an odd number of experimental conditions. Whereas complete pairwise balance can be achieved with a single square for  $n$  even, this cannot be done in the case of  $n$  odd. For example, it is impossible to achieve pairwise priority for  $n$  odd since the proportion of times Condition  $x$  precedes Condition  $y$  can never be exactly .5. In this case, two squares must be used to achieve design-wide pairwise balance. This can be done by selecting any first row from Table 3 and combining it with a square based on the reverse of the selected first row.

One implication of using two squares for  $n$  odd is that the minimum number of subjects required for full counterbalancing increases from  $n$  to  $2n$ . This suggests that there is a distinct advantage to employing repeated measures designs in which the total number of conditions is even. Thus, if the minimum number of

conditions needed to test the experimental hypothesis is odd, the researcher is urged to consider the benefits of adding one theoretically relevant condition. This would result in increased economy in terms of the minimum number to subjects required.

When the main experimental concern is with trends over a parametric independent variable (e.g., set size or exposure time), adding one more condition will also allow for a test of an additional, higher order orthogonal polynomial. When the repeated measures are the result of a factorial design (e.g., set size by exposure time by familiarity of word type), it is necessary that only one factor have an even number of levels. Conditions under which odd squares should still be used occur when an increase to an even design would result in excessive expense or excessive running time for subjects.

#### Statistical Considerations

Designs reported in this paper are balanced for additive residual or carry-over effects of the immediately preceding treatment. It should be noted that the residual effects may be more complicated; for example, multiplicative effects or those persisting beyond the immediately preceding treatment. If the structure of the residual is of some more complicated sort, these designs or any other Latin Squares may not be appropriate. The investigation of such residual effects is beyond the scope of the present paper.

Latin square designs were originally developed to deal with residual effects that are additive. To this point, the primary concern has been to control for the additive effects of serial

position. Conventional statistical analyses routinely include tests of significance for this factor. (It should be noted that these serial position effects are usually not of interest in psychological research and in practice are rarely even reported.) A unique feature of pairwise balanced Latin Squares is that they make it possible to statistically estimate the contribution of additive residual effects due to pairwise priority. To do this, one must use an analysis proposed by Williams (1949) and illustrated in Cochran and Cox (1957).

Perhaps obviously, a standard statistical analysis of a Latin Square could be used in the present case, assuming pairwise residual effects exist, and that treatments and residual effects are uncorrelated, then there will be a positive bias in the mean square for treatments (i.e., the mean square will be larger than in such residual effects didn't exist), and estimates of treatment effects will be confounded with residual effects. However, since the design is balanced, treatment effects will be confounded with their own residual effects. In contrast, Latin Square designs that are not pairwise balanced will have treatment effects which may be confounded with residual effects of other treatments. Further, there will also be a positive bias in the mean square for treatments, the extent of which will depend on the particular design, but which in general will be greater than that associated with the pairwise balanced designs.

Thus, if it is desirable to estimate the test direct treatment

effects and residual effects separately, the analysis given in Cochran and Cox (1957) is recommended. Note that this analysis applies strictly to pairwise balanced designs, and not to other designs in which carry-over may be suspected. Alternatively, a standard analysis of Latin Square designs, such as given in most textbooks, would be testing, in effect, the significance of an additive combination of treatments and their residual effects when applied to the pairwise balanced designs.

#### Computational Procedures

The computational procedures for pairwise balanced designs were first described by Williams (1949, 1950), and later reported in slightly modified form by Cochran & Cox (1957). Since neither source is commonly available to psychologists, we will present the Cochran & Cox notation, and illustrate its use with an example.

Designs differ in terms of whether more than one square is used and whether more than one subject is assigned to each row of the square. Normally, two or more squares will be used when  $n$  is odd. But also, it will sometimes be advantageous to use several squares in the case of  $n$  even. It will increase error degrees of freedom and can allow for greater control over the

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Insert Table 4 about here

---

distribution of pairwise distances. We have selected an example employing two 3x3 squares with one subject per row.

The analyses presented here assume that row, column, and treatment effects do not interact, and further, that the residual effect simply adds to the effect of the following treatment. Thus, for a row in which B follows A, and C follows B (i.e., treatment order A B C), the period in which C is applied has a total effect attributable to treatments which is  $(t_c + r_b)$ . Similarly, the total observed effect of treatments when B is applied would be  $(t_b + r_a)$ .

To simplify the presentation, let us assume that an experiment involves responses to three attitude statements, A, B, C, all presented to each subject. A given subject responds to a row of an appropriately selected square, and a column corresponds to the position in the order of presentation. Two squares are selected to be pairwise balanced. The squares are given in Table 4. Included in Table 4 is the hypothetical data, with one observation per cell.

The following symbols are used:

n = number of treatments (=3)

m = number of squares (=2)

S = total of sequence (row)

T = treatment total

P = total of position (column) in a given square

R = total of scores in positions immediately following the treatment in question

F = total of sequences (rows) in which this treatment is the last one

# Pairwise Balanced Latin Squares

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$P_1$  = total of scores in first position of all sequences

$G_j$  = grand total of scores in a given square

$G$  = grand total of all scores

The following quantities special to this analysis are needed:

$$P_1 = nG$$

$$nP_1 = (n + 2)G$$

Then some of the usual quantities are needed:

$$\text{Correction factor: } C = \frac{G^2}{mn^2}$$

$$\text{Total SS} = \sum X^2 - C \quad (\text{df} = mn^2 - 1)$$

$$\text{Sequences SS} = \frac{1}{n} \sum S^2 - C \quad (\text{df} = mn - 1)$$

$$\text{Positions (Order) SS} = \frac{1}{n} \sum P^2 - \frac{1}{n} \sum G_j^2 \quad (\text{df} = m(n - 1))$$

Note that this is a sum of squares between positions within squares.

$$\text{Treatments (unadjusted) SS} = \frac{\sum T^2}{mn} - C \quad (\text{df} = n - 1)$$

In addition, adjusted (for residual or carry-over) treatment effects are computed as follows:

$$\hat{T} = (\underline{n}^2 - \underline{n} - 1) \underline{T} + \underline{nR} + \underline{F} + \underline{P}_1 - \underline{nG}$$

and the adjusted sum of squares for treatments is computed:

$$SS_{\text{trts(adj)}} = \frac{\Sigma \hat{T}^2}{\underline{mn} (\underline{n}^2 - \underline{n} - 1) (\underline{n}^2 - \underline{n} - 2)} \quad (\text{df} = \underline{n} - 1)$$

Similarly, adjusted residual (carry-over) effects are computed:

$$\hat{R} = \underline{nT} + \underline{n}^2 \underline{R} + \underline{nF} + \underline{nP}_1 - (\underline{n} + 2) \underline{G}$$

and

$$SS_{\text{res(adj)}} = \frac{\Sigma \hat{R}^2}{\underline{mn}^3 (\underline{n}^2 - \underline{n} - 2)}$$

The total sum of squares for treatment effects (i.e., direct plus residual) is given by:

$$SS_{\text{trt}} = SS_{\text{trts(unadj)}} + SS_{\text{res(adj)}}$$

$$\text{or} = SS_{\text{trts(adj)}} + SS_{\text{res(unadj)}}$$

Three of these four quantities were computed above; the fourth,

$SS_{\text{res(unadj)}}$  may be computed by subtraction. However, to provide a check on computations,  $SS_{\text{res(unadj)}}$  may be computed directly:

First, for each treatment compute

$$\hat{R}' = \hat{R} + \underline{G} - \underline{nT}$$

Then the sum of squares is given by

$$SS_{res(unadj)} = \frac{\sum \hat{R}_i^2}{mn^3(n^2 - n - 1)}$$

Finally, SS error is obtained by subtraction from the total sum of squares. Here the sum of squares for Positions within Squares has been removed. If instead the overall sum of squares for Positions is removed (i.e., assuming no differences in Positions effects across squares), the error degrees of freedom becomes  $(n - 1)(mn - 3)$ .

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Insert Tables 5 and 6 about here

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Table 5 summarizes the computations using the formulae that are presented above. The analysis of Variance Summary Table is contained in Table 6. Note that the terms in brackets have the same total. Tests on the effects of treatments and residuals should only be made on the adjusted sums of squares. The Error sum of squares is obtained by subtraction of the total of all other non-redundant sums of squares from the Total, i.e.;

$SS_{tot} = 182.0$ . Thus, only one of the sums in brackets is unsolved in this subtraction, otherwise effects attributable to treatments (direct and residual) would be counted twice. Since  $F_{2,4} = 10.65$  for  $\alpha = .025$ , both direct treatment and residual effects are significant in this example.

This example does not involve a design in which a between Ss treatment effect is included, or in which multiple Ss are assigned to a row of a square. However, the extension of the analysis to such designs is straightforward and follows plans available in standard texts. The adjustment for carry-over (residual) involves adjustment of within subject effects. Further, computation of a Ss x Positions effect (the usual within Ss error) in this case would be unaffected. Between Ss effects would use the usual Between Ss error term.

References

- Alimena, B. S. (1962). A method for determining unbiased distribution in the latin square. Psychometrika, 27, 315-17.
- Campbell, D. T., & Stanley, J. C. (1966). Experimental and quasi-experimental designs for research. Chicago: Rand McNally.
- Carlsmith, J., Ellsworth, P., & Aronson, E. (1976). Methods of research in social psychology. Reading, Mass.: Addison-Wesley.
- Christensen, L. B. (1980). Experimental Methodology (2nd Edition). Boston, Allyn & Bacon.
- Cochran, W. G., & Cox, G. M. (1957). Experimental designs (2nd Edition). New York: Wiley.
- Crano, W. D., Brewer, M. B. (1973). Principles of research in social psychology. New York: McGraw-Hill.
- D'Amato, M. R. (1970). Experimental psychology methodology: Psychophysics and learning. New York: McGraw-Hill.
- Denes, J., & Keedwell, A. D. (1974). Latin squares and their applications. New York: Academic Press.
- Fisher, R. A., & Yates, F. (1953). Statistical tables for biological, agricultural, and medical research (4th Edition). Edinburgh: Oliver & Boyd.
- Greenwald, A. G. (1976). Within-subjects designs: To use or not to use? Psychological Bulletin, 83, 314-320.
- Isaac, P. D., McCann, C. D., & Ostrom, T. M. (1983). Generation of pairwise balanced Latin Squares. Unpublished manuscript,

Ohio State University, Columbus, Ohio.

- Lindman, H. R. (1974). Analysis of variance in complex experimental designs. San Francisco; W. H. Freeman & Co.
- Murphy, M. D., & Puff, C. R. (1982). Free recall: Basic methodology and analyses. In C. R. Puff (Ed.), Handbook of research methods in human memory and cognition. New York: Academic Press.
- Myers, J. S. (1979). Fundamentals of experimental design (3rd Edition). Boston: Allyn & Bacon.
- Poulton, E. C. (1973). Unwanted range effects from using within-subject experimental designs. Psychological Bulletin, 80, 113-21.
- Poulton, E. C. (1974). Range effects are characteristic of a person serving in a within-subjects experiment design -- a reply to Rothstein. Psychological Bulletin, 81, 201-2.
- Poulton, E. C. (1982). Influential companions: Effects of one strategy on another in the within-subjects designs of cognitive psychology. Psychological Bulletin, 91, 673-690.
- Rothstein, L. D. (1974). Reply to Poulton. Psychological Bulletin, 81, 199-200.
- Wagenaar, W. A. (1969). Note on the construction of diagram-balanced latin squares. Psychological Bulletin, 72, 384-6.
- Williams, E. J. (1949). Experimental designs balanced for the estimation of residual effects of treatments. Australian Journal of Scientific Research -- Series A, 2, 149-168.

Williams, E. J. (1950). Experimental designs balanced for pairs of residual effects. Australian Journal of Scientific Research -- Series A, 3, 351-63.

Winer, B. J. (1972). Statistical principles in experimental design. New York: McGraw-Hill.

Table 1  
 Latin Square Selection Based on Traditional Criteria

Subject Type	Standard Square				Recommended Square			
	Treatment Order				Treatment Order			
	First	Second	Third	Fourth	First	Second	Third	Fourth
I.	0	1	2	3	3	0	2	1
II.	1	0	3	2	2	1	3	0
III.	2	3	1	0	1	3	0	2
IV.	3	2	0	1	0	2	1	3

Note - Based on Winer (1972, p. 689).

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### First Rows of Pairwise Balanced Squares for n Even

$\frac{0}{0} \mid n = 4$															
	1	3	2												
$\frac{0}{0} \mid n = 6$															
	1	5	2	4	3										
	2	1	4	5	3										
$\frac{0}{0} \mid n = 8$															
	1	7	2	6	3	5	4								
	1	3	6	2	7	5	4								
$\frac{0}{0} \mid n = 10$															
	1	9	2	8	3	7	4	6	5						
	1	8	2	4	9	7	3	6	5						
	4	1	3	2	7	8	6	9	5						
	9	6	8	2	7	3	1	4	5						
	6	4	7	8	3	2	9	1	5						
$\frac{0}{0} \mid n = 12$															
	1	11	2	10	3	9	4	8	5	7	6				
	7	11	8	10	9	3	4	2	5	1	6				
	8	1	4	5	3	9	11	10	7	2	6				
	2	1	10	5	9	3	1	4	7	8	6				
	1	4	2	9	5	11	3	8	10	7	6				
$\frac{0}{0} \mid n = 14$															
	1	13	2	12	3	11	4	10	5	9	6	8	7		
	6	1	5	2	4	3	10	11	9	12	8	13	7		
	13	8	12	9	11	3	10	4	2	5	8	6	7		
	8	6	9	5	10	11	4	3	12	2	13	1	7		
$\frac{0}{0} \mid n = 16$															
	1	15	2	14	3	13	4	12	5	11	6	10	7	9	8
	9	15	10	14	11	13	12	4	5	3	6	2	7	1	8
	13	15	14	2	11	1	12	4	9	3	10	6	7	5	8
	5	15	6	2	3	1	4	12	9	11	10	14	7	13	8

**Note. - n = number of experimental treatments**

## 25

### First Rows of Pairwise Balanced Squares for $n$ Odd

[illegible]

**Note. - n = Number of experimental treatments.**

# Pairwise Balanced Latin Squares

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Table 4

## Pairwise Balanced 3x3 Latin Squares and Simulated Data

### Latin Squares

<u>Square 1</u>				<u>Square 2</u>			
Treatment Position				Treatment Position			
<u>Subject Type</u>	<u>First</u>	<u>Second</u>	<u>Third</u>	<u>Subject Type</u>	<u>First</u>	<u>Second</u>	<u>Third</u>
I	0	1	2	IV	2	1	0
II	1	2	0	V	0	2	1
III	2	0	1	VI	1	0	2

### Simulated Data

<u>Square 1</u>				<u>Square 2</u>			
$\Sigma$				$\Sigma$			
-4	-6	1	-9	6	7	-2	11
1	1	0	2	-4	-2	2	-4
3	2	-1	4	0	1	1	2
$\Sigma$	0	-3	0	$\Sigma$	2	6	9

Table 5  
Summary of Computations

Treatment Number	<u>T</u>	<u>R</u>	<u>F</u>	<u>T̂</u>	<u>t̂=T̂/24</u>	<u>R̂</u>	<u>r̂=R̂/24</u>	<u>k̂'</u>
0	-7	-8	13	-62	-2.58	178	-3.25	-51
1	3	1	0	2	.08	-6	-.25	-9
2	10	11	-7	60	2.50	84	3.5	60

$$\underline{P}_1 - \underline{nG} = 2 - 3(6) = -16$$

$$\underline{nP}_1 - (\underline{n} + 2)G = 3(2) - (3 + 2)6 = -24$$

$$\hat{T} = 5\underline{T} + 3\underline{R} + \underline{F} - 16$$

$$\hat{R}' = 3\underline{T} + 9\underline{R} + 3\underline{F} - 24$$

$$\underline{C} = \frac{\underline{G}^2}{2 \cdot 3^2} = \frac{36}{18} = 2.0$$

$$SS_{\text{tot}} = 2X^2 - \underline{C} = 184 - 2 = 182$$

$$SS_{\text{seq}} = \frac{1}{3} \{ \Sigma \underline{S}^2 \} - \underline{C} = \frac{1}{3} \{ (-9)^2 + (2)^2 + (2)^2 \} - 2 = 78.67$$

$$SS_{\text{pos w/seq}} = \frac{1}{3} \{ \Sigma \underline{P}^2 \} - \frac{1}{3} \{ \Sigma \underline{Gj}^2 \}$$

$$= \frac{1}{3} \{ 0^2 + (-3)^2 + \dots + (1)^2 \} - \frac{1}{9} \{ (-3)^2 + (9)^2 \} = 6.67$$

$$SS_{\text{trt(unadj)}} = \frac{\Sigma \underline{T}^2}{2(3)} - \underline{C} = \frac{1}{6} \{ (-7)^2 + (3)^2 + (10)^2 \} - 2 = 24.33$$

# Pairwise Balanced Latin Squares

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$$SS_{\text{trt(adj)}} = \frac{\hat{\Sigma T}^2}{6(5)(4)} = \frac{(-62)^2 + (2)^2 + (60)^2}{120} = 62.07$$

$$SS_{\text{res(adj)}} = \frac{\hat{\Sigma R}^2}{2 \cdot 3^3 (3^2 - 3 - 2)} = \frac{(-78)^2 + (-6)^2 + (84)^2}{216} = 61.00$$

$$SS_{\text{res(unadj)}} = \frac{\hat{\Sigma R'}^2}{2 \cdot 3^3 (3^2 - 3 - 1)} = \frac{(-51)^2 + (-9)^2 + (60)^2}{270} = 23.27$$

Pairwise Balanced Latin Squares

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Table 6

Analysis of Variance Summary Table

Source	df	SS	MS	F
Sequences	$\underline{mn} - 1 = 5$	78.67	15.73	
Positions within squares (cols)	$\underline{m}(n - 1) = 4$	6.67	1.67	
Direct Treatment and Residual				
Treatment effects (unadj)	$\underline{n} - 1 = 2$	24.33	12.17	
Residual effects (adj)	$\underline{n} - 1 = 2$	61.00	30.50	10.78
Residual effects (unadj)	$\underline{n} - 1 = 2$	23.27	11.64	
Treatment effects (adj)	$\underline{n} - 1 = 2$	62.07	31.04	10.97
Error	$(\underline{n} - 1)(\underline{mn} - \underline{m} - 2) = 4$	11.32	2.83	
Total	$\underline{mn}^2 - 1 = 17$	182.0		

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